10
Extending the Geometry to Systems of Fractal Cities

. . . in the final analysis, fractal methods can serve to analyze any 'system', whether natural or artificial, that decomposes into 'parts' articulated in a self-similar fashion, and such that the properties of the parts are less important than the rules of the articulation. (Mandelbrot, 1983, p. 114.)

10.1 Articulating Systems of Cities

Throughout most of this book, our concern has been with using fractal geometry to describe and model the shape and distribution of population within individual cities. In Chapter 6, we focussed upon treating individual land uses as fractal objects, and examining how fractal geometry could be used to infer the dimensional properties of the entire distribution of land use shapes. In Chapter 1, we alluded to the manner in which the spatial hierarchy of cities gave rise to a rank-size distribution, but as yet, we have not explored how this geometry might be extended to entire systems of cities. This will be the quest of the present chapter. There is of course a well-worked-out theory of city size known as central place theory which we referred to in Chapter 1 (Christaller, 1933, 1966) and to which we must relate our extensions of this geometry. Just as we articulated a city in terms of a hierarchy of development and free space using the Sierpinski carpet model in Chapters 2 and 7, it is possible to generate a hierarchy of cities, beginning with a primate city as generator and then partitioning its hinterland or sphere of influence successively, generating a distribution of city sizes and frequencies from the largest to the smallest. In central place theory, the space of the largest city and its hinterland is first exhaustively partitioned into a series of equal and lesser-sized hinterlands, which in turn are subdivided into lower levels of hierarchy, thus generating a size distribution often referred to as the rank-size rule. From considerations relating to the optimal packing of hinterland shapes, the hexagonal hinterland area emerges, and in terms of these shapes, a nested set of hexagonal market areas is the result. Various size distributions can be generated depending upon the partitioning used, while overlapping hinterlands are also possible as we illustrated in Chapter 1.

To illustrate this idealized system, assume that the largest hinterland, which will be our starting point, has a linear measure L_0. Now let us define a scaling ratio ε_k which when applied to the original measure L, generates a measure of the size of a lower order hinterland L_k where k is the order or rank of the city and its hinterland in question:

$$L_k = L_0\ \varepsilon_k. \tag{10.1}$$

Assume that the hinterland of the largest city associated with $k = 0$ is square, that is, its field area $U = L_0^2$, that each successive hinterland associated with the partitioning is also square, and that its subdivision ratio from level to level is given as r. Note that this ratio must be positive but less than 1, for it must yield a smaller measure L_k when applied to L_{k-1}, that is

$$L_k = L_{k-1}r. \tag{10.2}$$

Moreover, through recursion from L_0, equation (10.2) can be written as

$$L_k = L_0 r^k, \tag{10.3}$$

where it is now clear that $\varepsilon_k = r^k$ in equation (10.1).

Because the original hinterland is assumed to be square, the number of cities and their hinterlands generated by this process is given as

$$n_k = [(r^k)\ (r^k)]^{-1} = r^{-2k}, \tag{10.4}$$

and the area U_k of each city hinterland in this grid-based hierarchy is thus

$$U_k = L_k^2 = L_0 r^{2k} = L_0 n_k^{-1}. \tag{10.5}$$

A couple of examples illustrate the typical size distribution which can be generated. If we assume that the original area L_0 (= 1 unit of measure) is subdivided into four subspaces at the first level of hierarchy implying that $r = 1/2$, then the number of cities generated from (10.4) are 1, 4, 16, 64, 256, and so on, with their associated linear dimensions as 1, 1/2, 1/4, 1/8, 1/16 ... and their areas as 1, 1/4, 1/16, 1/64, 1/256 .... If the subdivision were 1/3 as in the Sierpinski carpet (see Chapter 2 or Chapter 7), then the frequency distribution would be 1, 9, 81, 729, 6561, and so on. Of course, it would be possible to generalize this process using a non-square space such as the hexagon and also a packing parameter which did not assume square subdivision, that is generalizing equation (10.4) as $n_k = r^{-\gamma k}$, where γ is a now parameter of the system in question. But these are details which we do not have time to pursue here, nor are they essential to our quest.

The most important assumption which we will make, however, relates to the populations which are associated with this system of generating cities. We will in fact assume that the population of each city in the hierarchy is generated within its space using a DLA-like process which assumes that population N_k scales with the linear size of its space L_k according to the fractal dimension D. Using this notation our classic scaling relation which we have previously specified in equations (2.32), (7.6) and (9.31) is

$$N_k = \varphi L_k^D, \tag{10.6}$$

where $1 < D < 2$. Then from equation (10.3) or (10.5)

$$N_k = \varphi L_0^D r^{Dk} = \varphi N_0 r^{Dk}. \tag{10.7}$$

where N_0 is the population of the primate city in the hierarchy. If we now note that r can be written as $1/z$ where z is the number of additional cities generated from one level of the hierarchy to the next, then

$$N_k = \varphi \frac{N_0}{z^{Dk}}. \tag{10.8}$$

Equation (10.8) can be considered as a generalized rank-size equation in which population size is solely a function of rank, assuming that the parameter D is constant. The only way, however, that we are able to generate a strict rank-size rule of the form $N_k = \varphi\ N_0/k^{\tau}$ from equation (10.8) is by assuming that the fractal dimension D varies with rank, that is that $D = (\log k)/k$. This implies that as city size increases, the fractal dimension also increases in value. In turn, this implies that the density parameter α would decrease in value with city size and this would appear to be mildly consistent with some empirical evidence (Clark, 1951; Mills, 1970; Mogridge, 1984), although the question remains ill-defined. In fact, although this analysis is highly suggestive of the way we might connect up central place theory to fractal geometry and to urban density functions, its implications are well beyond what we are able to pursue in this book and must await further sustained research. The analysis, however, is rich with implications for the way we might begin to fuse intra- and inter-urban theory, theories of what happens inside the city with those which seek to show how systems of cities develop. As such, it represents a major direction for future work in human geography and urban economics.

What we have begun to sketch here is a basis for a preliminary exploration of the relationship between population size and linear dimension over a system of cities. What we have not yet examined is the possibility that the fractal dimension might actually vary systematically over this size distribution for this is something we wish to first test. The city size distributions generated here like those we generated earlier within individual cities using the Sierpinski carpet model, are based on a top-down approach, and there are no implications for how cities might actually change their position within the hierarchy through growth or decline. We will, in fact, assume that the fractal scaling laws governing the population and its distribution within the individual city, can be extended in a straightforward way to a system of cities as we have already adopted in equations (10.6) to (10.8) above. If the traditional rank-size rule were an accurate portrayal of city size distribution, then this would imply that D would increase in value as cities grew, but we consider that these speculations are so uncertain and the models postulated no more than examples of the fractal approach, that we have confidence in proceeding by assuming the constancy of D.

Before we begin to show how our theoretical model might be tested on different systems of urban settlements, we need to note its relation to the concept of allometry which we alluded to in earlier chapters. Allometry, according to Gould (1966), is used "to designate the differences in proportions correlated with changes in absolute magnitude of the total organism or of the specific parts under investigation". More commonly, the term is used to describe scaling relations between two 'size' measures of an organism or system under study (Mark and Peuker, 1978). One relation linking perimeter length of an urban boundary to its area was stated in

Chapter 2 as equation (2.29) and used extensively in Chapter 6. In Chapter 9, the relation between population and the size of its urban field was also examined in equations (9.30) and (9.31). But a much stricter and more conventional view of the allometric relation between population and its actual area of occupation will be developed here. From equations (10.6) and (10.7), it is clear that the area occupied by the population – its development – must vary with the population itself assuming that the density of occupancy of the elemental unit is the same, regardless of city size. Thus $N_k \propto A_k$ where A_k is the occupied, developed or built-up area. This is the relation that we will also test in the sequel with a view to determining whether the city system displays positive or negative allometry, or even isometry. On this basis we might speculate as to whether cities truly grow into their third dimension or not.

A related theme in this chapter concerns the measurement of size and shape, area and density. In particular, we will make a central distinction between the concepts of the built-up urban area and the urban field, focussing upon the need to relate the particular measurement in question to the purpose of the analysis. It is already very clear to us from the literature on the measurement of urban density reviewed in Chapter 9 that conventional practice is confused, and our confidence in previous empirical estimates of allometric and other scaling relationships in urban studies is low. Another theme, but one which is of different import, involves the representation of spatial shape and area in computer models and information systems which are concerned with spatial manipulation, analysis and display. Thus our models which are based on describing size and shape, also have some more practical implications for the representation of digital data.

In this chapter then, we will work towards a consistent theory of urban growth and form in a system of urban settlements, combining allometric relationships and fractal geometries. We will illustrate our theory with data on the size, shape and spacing of urban settlements in two case studies: in the County of Norfolk in the English region of East Anglia, and then in the whole of the South East region of England where we will explore the extent to which the growth of the settlement pattern has been constrained by planning policies, specifically those instruments of early 20th century planning known as 'green belts'. We will introduce a standard data base for both case studies, briefly reviewing the principal means by which urban shapes and areas are represented through boundaries or 'envelopes'. We will then apply the various scaling relationships which we consider of major importance in linking size to shape through dimension, to the urban settlement system in Norfolk, validating the hypotheses which we will set out in the next section. We will then present a more refined but more speculative analysis using the same scaling relations, attempting to classify settlements according to their various dimensions with a view to determining whether some settlements have been affected by explicit planning policies in terms of their size and shape. In this way, we will conclude our introduction to the fractal city by showing how we might use the geometry presented here to inquire into the impact of ongoing planning policies and other forms of public decision-making.

10.2 Scaling Relations for City Size Distributions

The two basic measures of size which we will use are population and area. Our task will be to seek relationships between these variables, first by researching how these variables might best be defined, and second, by exploring how the scaling model of the previous section might be used to illuminate the postulated relations. In the rest of this chapter, we will explore these relations first using data from the pattern of urban settlement in the English County of Norfolk in the region of East Anglia, and then with a much wider set of the same data for the whole of South East England. Associated with the population N_k of any urban cluster k, there might be several definitions of area. Note that as each cluster is of a different size, and if these are ordered by population, then the index k, in fact, is consistent with rank-size.

We will use two distinctive measures of area here: first there is the occupied area called A_k which can loosely be defined as the built-up or developed area, and is likely to covary to an extent with population. Second, there is the urban field whose area U_k can be defined as the hinterland immediately associated with the greatest radial extent of the cluster (Hagerstrand, 1952). This may be the immediate circle of area within which growth has already taken place, or as in the theoretical model in the previous section, the square area defined by equation (10.5). There is also a fourth variable of interest which relates area A_k to field size U_k, and this is the urban envelope E_k defined as the length of the boundary or perimeter which marks the greatest extent of the built-up area, and which we used extensively in Chapters 6 and 7 in our early forays into the geometry of the fractal city. To provide some meaning to these concepts, we have illustrated their spatial definition using the example of the largest town from our data set, Norwich; these definitions are shown in Figure 10.1.

Figure 10.1(a) shows the built-up urban area whose extent A_k is indicated by the cross hatch, and it is this area that contains the population N_k. The urban field is shown in Figure 10.1(b), and this is the bounding circle based on the center of the cluster, marked by the maximum radius R_k which contains the whole cluster. The area of the cluster is given as $U_k = \pi R_k^2$ and $U_k > A_k$. The urban envelope is shown in Figure 10.1(c), its length E_k being a measure of both the size and the shape of the cluster. In Figure 10.1(d), the maximum spanning distance across the cluster – 'Feret's diameter' – was defined earlier in equation (5.12) (Kaye, 1989b); the length of this span is defined as F_k, and this will be used later in estimating and approximating the radius R_k. Note that in the sequel, we will use the radius R_k in preference to the linear measure L_k introduced earlier. We will examine two types of relationship between these variables, first relating population N_k to area A_k and to field radius R_k, second relating the length of the envelope E_k to these same variables. These types of relationship are central to allometry or 'relative size' relationships (Gould, 1966), and by relating size and length to area, this enables us to explore questions of density. In this way, we are able once again to relate our work to the mainstream literature on urban allometry (Dutton, 1973) which we have already introduced in Chapters 7 to 9 in the study of urban population density and form. Here, however, our

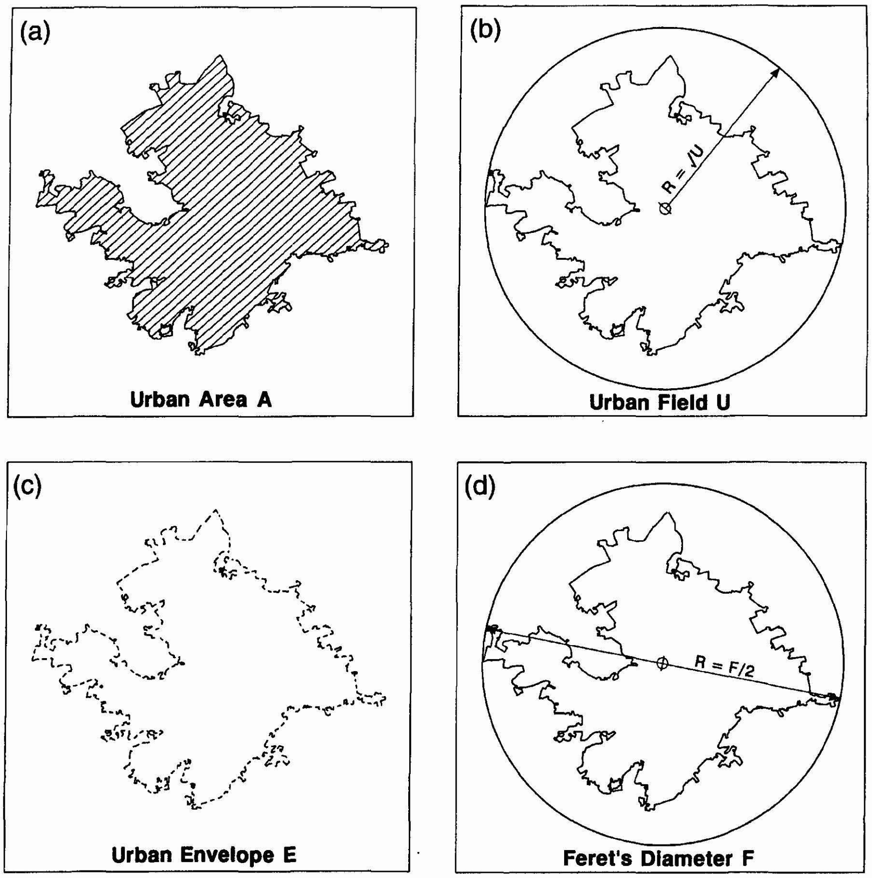

Figure 10.1. Definitions of urban area, field, envelope and radius.

use of allometric relations will be more conventional, with our emphasis on fitting such relations to sets of different sized objects – towns and cities in this case, in contrast to our previous use of these relations for examining changes in the size of the individual city.

The classic allometric relation we will begin with involves the relationship between population size N_k and occupied area A_k which we can write as

$$N_k = \gamma A_k^{\beta} = \gamma A_k^{\Delta/2}. \tag{10.9}$$

γ is a constant of proportionality and β is a scaling constant. In equation (10.9), we have also written β as $\Delta/2$ where Δ can be interpreted as a 'dimension' of the occupied area, scaling the radius R_k of such an area ($R_k = A_k^{1/2}$) to population. The use of this convention will become clear in the sequel when all the scaling parameters have been introduced. As we pointed out in Chapter 9, there is obviously a strong relationship between population and area, although the precise form of the scaling is problematic. Nordbeck (1965, 1971) suggests that the scaling constant β should be 3/2 using the argument that population growth takes place in three dimensions; thus if $R_k = A_k^{1/2}$ is taken as the linear size of area, then $N_k =$

$\gamma R_k^3 = \gamma A_k^{3/2}$. This hypothesis is borne out in an analysis of the urban population of Sweden in 1960 and 1965 (Nordbeck, 1971). Results from urban density theory also suggest that as cities get bigger, their average density increases but the empirical evidence on this is mixed and is much complicated by the definitions of urban area used (see Muth, 1969). However, Woldenberg (1973) shows quite unequivocally that $\beta \approx 1$ from an analysis of two large population–area data sets for American cities.

In the case of the scaling model introduced earlier, it is clear that the area occupied by the population N_k varies as the population itself. This point was also made in equation (7.7) where the same analysis was applied to the individual city. For the growing fractal, the area of each occupied cell is assumed to be identical, say ε^2, thus the total urban area is $A_k = N_k\varepsilon^2$. In short, the population density $N_k/A_k = \varepsilon^{-2}$ is constant regardless of scale or the stage reached in the growth process. In summary then, we might expect the empirical relation between N_k and A_k to be of the simplest kind – perfect scaling – with both the theoretical model and much empirical evidence suggesting that $\beta \approx 1$ and $\Delta \approx 2$.

With respect to the urban field, the scaling between N_k and U_k is more complicated. As cities grow, their field becomes correspondingly larger, growing at a more than proportionate rate, and in the case of very large cities, the urban field is often considered to be global. This implies that as cities grow, their field density N_k/U_k always decreases. As in previous chapters, it is more appropriate to represent the field area U_k in terms of its radius $R_k = U_k^{1/2}$. Thus the field relationship can be stated as

$$N_k = \varphi R_k^D = \varphi U_k^{D/2}. \tag{10.10}$$

φ is a constant of proportionality and D is the scaling constant, the fractal dimension which will be less than 2 but always greater than 1 as can be seen from Figure 10.1(b). In terms of the idealized central place theory model of the previous section, equation (10.10) is equivalent to equation (10.6) and this is the basic scaling relation linking population to the size of its city which we used earlier in equations (2.32), (7.6) and (9.31).

Relationships between the length E_k of the bounding envelope of urban development and the area A_k and field radius R_k will also be explored here. It is important to note that the bounding envelope is not the perimeter of the cluster in that any undeveloped interior of the cluster is not detected by the envelope (see Figure 10.1(a) and (b)). In fact, the perimeter of a DLA cluster varies directly with its population as we indicated in Chapter 7 (see equation (7.7)). As the envelope defines the outer edge of the cluster, it is likely to be smoother and less circuitous than the perimeter, and this suggests that any measure of the fractal dimension of such a line is likely to be less than the fractal dimension of the cluster. In the case of the urban area A_k, we can relate the envelope to the assumed radius $R_k = A_k^{1/2}$ of occupied area, giving

$$E_k = \zeta A_k^{\omega} = \zeta A_k^{\delta/2}, \tag{10.11}$$

while for the field radius, a similar relation is postulated:

$$E_k = \nu R_k^{\check{D}}, \tag{10.12}$$

where ω, hence δ in equation (10.11) and $\check{D}$ in equation (10.12) can be regarded as 'dimensions' with ζ and ν as constants of proportionality.

Before we summarize the relationships which we seek to validate empirically, it is worth noting the theoretical bounds within which our analysis will take place. It is clear that cities with a variety of forms of development from the linear to the compact circular are consistent with the theoretical model we outlined in the first section. In the case of the completely compact cluster, its occupied area and its field are coincident with $N_k = \gamma A_k = \varphi U_k \propto \pi R_k^2$ and with $\Delta = D = 2$. The growing zone at the edge of the cluster is the same as the perimeter, and this is defined as the derivative of N_k with respect to radius R_k, that is $dN_k/dR_k \propto \pi R_k$. The envelope is also the perimeter in this case with $E_k = \zeta A_k^{1/2} = \nu R_k \propto \pi R_k$ and $\delta = \check{D} = 1$. In the case where the cluster is linear $N_k = \gamma A_k^{1/2} = \varphi U_k^{1/2} \propto \pi R_k$ and $\Delta = D = 1$, while the derivative of N_k does not provide the formula for the perimeter, just the growing zone which is always a point of zero dimension, implying in this case that $\delta = \check{D} = 0$. In the case of a real urban cluster which does not completely fill its available space, area, perimeter and envelope can be approximated by space-filling lines which suggest that all the dimensions of significance – Δ, D, $\check{D}$, and δ – will be between 1 and 2. The only examples we are aware of where the dimensions of urban envelopes have been estimated are those we illustrated in Chapters 5 and 6 which yielded values between about 1.1 and 1.5, in contrast to those for the population–radius relations which from Table 7.1 lie between about 1.5 and 1.9.

Pulling all these threads together, we will hypothesize that the four dimensions associated with the four scaling relationships given in equations (10.9) to (10.12) should be ordered as $1 < \check{D} < \delta < D < \Delta$, where $\check{D}$, $\delta \approx 1.26$, $D \approx 1.71$ and $\Delta \approx 2$. The constants associated with these four relationships can be estimated from regressions of their log-linearized forms. We will refer to these relationships as being of allometric or DLA (diffusion-limited aggregation) type, involving independent variables of occupied area or urban field. The log-linearized forms of equations (10.9) to (10.12) are given as

$$\log N_k = \log \gamma + \beta \log A_k, \ (\Delta = 2\beta), \tag{10.13}$$

$$\log N_k = \log \varphi + D \log R_k, \tag{10.14}$$

$$\log E_k = \log \zeta + \omega \log A_k, \ (\delta = 2\omega), \tag{10.15}$$

$$\log E_k = \log \nu + \check{D} \log R_k. \tag{10.16}$$

Equations (10.13) to (10.16) will be those whose parameters will be estimated in the sequel and used to establish the consistency between the form of the urban settlement systems in Norfolk, and in South East England, and the theoretical allometric and DLA relationships outlined in this and the previous section.

10.3 The Representation of Urban Areas

We have already focussed upon some of the difficulties of measuring the relationship between the size and form of urban settlements. Early work on the size relations within settlement systems was necessarily restricted by the quality of the measures of the precise extent and population size of constituent areas. Naroll and von Bertalanffy (1956) attributed much of the variation in international urban–rural population ratios to differing national definitions of 'urbanity' and the differing areal extent of data collection units which together comprise urban areas. Newling (1966) encountered problems of the changing areal basis of data collection in his study of the evolution of intra-urban population density gradients over time. And as we have noted, Woldenberg (1973) obtained some quite radically different estimates of population–size relations in his cross-sectional study of the US settlement system, which depended upon his use of one or other of two atlases to source his urban area measurements. In the face of such vagaries and inconsistencies, it is scarcely surprising that the nature of the theoretical relationship between size and spatial form remains obscure. We have already begun to clarify some of these issues in earlier sections, and our empirical analysis which follows is designed to cast further light on these questions.

The causes of these discrepancies and sources of possible measurement errors are increasingly understood, and the routine innovation of digital databases holds the prospect of greater precision in the delineation of urban areas and monitoring of the areal impacts of change (Shepherd and Congdon, 1990). But nevertheless, there remains cause for concern that even in the data-rich environment of the 1990s, the effects of different measurements of areal units will go undetected in spatial analysis. Moreover, there exist acute definitional difficulties with respect to what is and what is not unambiguously 'urban', and the distance threshold beyond which outlying urban parcels should be classified as physically (and possibly, by extension, functionally) separate from main urban areas. Our own investigations in the examples used throughout this book using comparable boundary data recorded at different spatial scales, but based upon slightly different digitizing criteria, suggest that areal discrepancies of the order of 20% to 30% are likely to be quite common for most settlement sizes. Taken together, this makes it difficult to assess precisely how marginal increments in population lead to changed boundaries of urban forms through the process of accretion, and there is a clear need to develop stronger links between measurement and theory.

In this context however, all our data are represented in the theoretically more accurate vector mode. The data source used in both the Norfolk and South East England examples, is the Office of Population Census Statistics (OPCS) urban areas data base (OPCS, 1984) in which urban areas are defined as follows: land on which permanent structures are situated; transportation corridors (roads, railways and canals) which have built-up sites on one or both sides, or which link built-up sites which are less than 50 m apart; transportation features such as railway yards, motorway service areas, car parks as well as operational airfields and airports; mineral workings and quarries, and any area completely surrounded by built-up sites. The areas were identified using the 1:10,560 Ordnance Survey series in conjunction with Population Census Enumeration District (ED) base maps. These maps were used to ascertain which areas of urban land contained four or more EDs, and on this basis, these qualified as urban areas.

Population figures from EDs which had 50% or more of their population within an urban area were included in the population total for that area. Further general information and details of the treatment of small areas of population and discontiguous urban land can be found in OPCS (1984). These boundaries were then reduced to the 1:50,000 scale and computer digitized to an accuracy of 0.5 mm permitting inaccuracies of up to 250 m on the ground. Our first case study uses data for the County of Norfolk, our second for the 13 counties composing the standard region of South East England, both of which have been extracted from this source.

10.4 Initial Analysis of the Norfolk Settlement Pattern

The data comprise 86 distinct urban settlements from populations as small as 45 to the major county town of Norwich which has about 186,000 people. The pattern and form of these urban settlements are shown in Figure 10.2. We have already alluded to the difficulty of defining and adhering to definitions of urban land which are both unambiguous and appropriate to any specific task, and it is likely that the original decision by OPCS to include some of the smallest settlements was in practice an arbitrary one. We anticipate that the population and area of these smallest settlements would not closely correspond to any empirical regularities extant elsewhere in the data set, as a result of disproportionate errors in the measurement of their populations and bounding envelopes. Settlements whose form is dominated by transportation infrastructure are also likely to be 'unusual' in both geo-

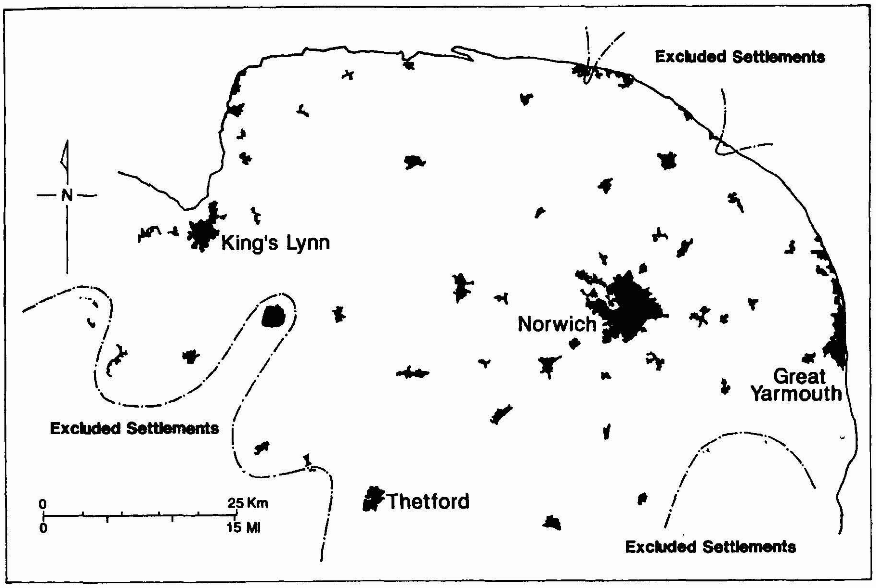

Figure 10.2. The pattern of urban settlement in Norfolk.

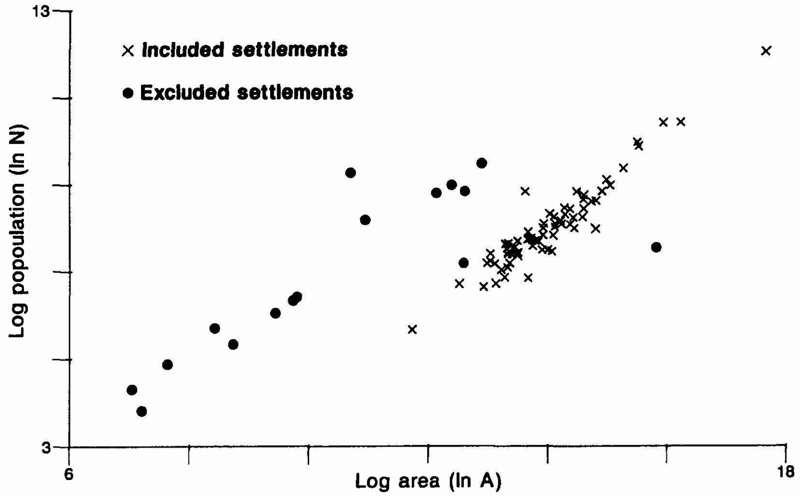

Figure 10.3. The population–area relation for the Norfolk settlement system.

metrical and population terms, with such settlements primarily, but not exclusively, small in size and scale.

Our theory posits that population and area covary in a systematic way, and thus our preliminary analysis began by assessing whether this was indeed the case. Figure 10.3 illustrates the relationship between population and urban area for the entire Norfolk settlement system. This figure depicts an unambiguous relationship across most of the range of settlement areas, although this relationship breaks down amongst the 15 smaller settlements. These settlements are shown in Figure 10.3 by the solid circles. Several related criteria were used for their exclusion: all 15 settlements are those which have less than 50 digitized pairs of coordinates defining their urban areas, thus making computation of their fractal dimensions unreliable using

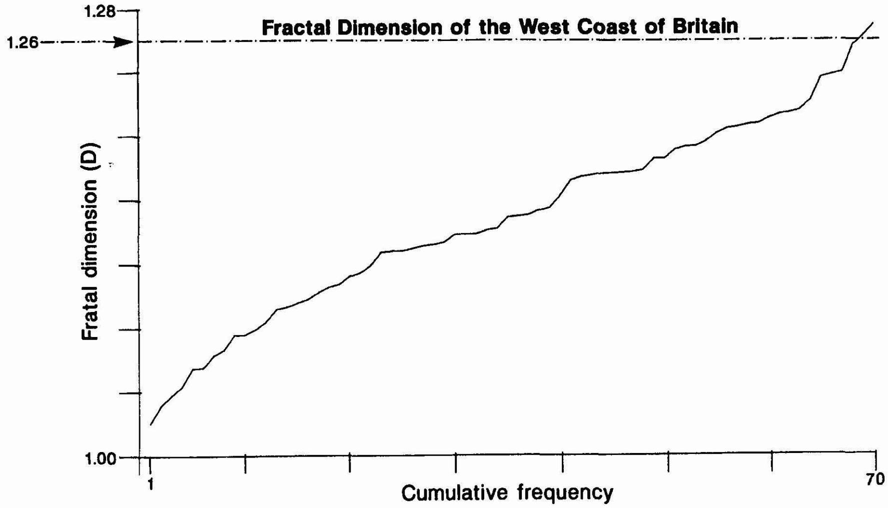

Figure 10.4. The cumulative distribution of fractal dimensions.

the Richardson (1961) 'walking dividers' method. These settlements were also amongst the smallest in terms of population and area, and are mainly located on the edge of the region. Some are cut by the regional boundary, hence form only parts of settlements, thus requiring their exclusion from the data set.

It is reasonable to anticipate this on *a priori* grounds since the form of small settlements is likely to be dominated by the transport network rather than by density–size relations. Although there is visual evidence to suggest that a different relationship holds for these smaller settlements, we nevertheless simply disregarded them in our subsequent analysis since our focus is upon the growth of settlements which might be unambiguously described as 'urban'. We can also note that the dominant population–area relation only appears to establish itself above a rough area threshold and thus suggests that this is a consequence of the dominant impact of transport infrastructure beneath this threshold. The single other settlement whose shape is unquestionably distorted by transport infrastructure is Marham Airfield. This 'settlement' has large area but low population and thus constitutes an outlier to the main relationship: as such it too was removed from the subsequent analysis which was based on the remaining 70 settlements.

We initially computed fractal dimensions for each of these 70 settlements. Calculation of such dimensions is now an established diagnostic for identifying the structure and character of digitized curves (Muller, 1986, 1987). The fractal dimensions of each individual settlement were first computed using the 'structured walk' algorithm based on Richardson's (1961) method of spanning each digitized curve at different scales and calculating their associated lengths. This algorithm which we first outlined in Chapter 5, entails measurement of the boundary envelope of each area at a range of successively finer scales, thus yielding correspondingly increased length measurements as more and more detail on the base curve is picked up. The range of scaled measurements obtained for each parcel was set at between half the mean digitizing intensity for that parcel and one-half of Feret's diameter, the maximum spanning distance between any two points on the digitized base curve (Kaye, 1989a), shown earlier in Figure 10.1(d) for Norwich. Regression analysis was then performed on the paired envelope–scale length points to establish whether the envelope is indeed fractal from the value of its (fractal) dimension. In Chapters 6 and 7, we found that the structured walk method is the most reliable and robust procedure for computing such dimensions.

Figure 10.4 illustrates the distribution of fractal dimensions for the subset of 70 settlements, in terms of their cumulative frequency, also indicating the fractal dimension of the west coast of Britain ($D \approx 1.26$) for comparison (Richardson, 1961). The mean value of our settlements is rather lower at 1.148 with a standard deviation of 0.059 and this would appear to reflect the less intricate nature of man-made boundaries. These dimensional measurements are not directly comparable with the other measurements reported below due to the fact that our subsequent analysis is based on computing fractal dimensions using the set of 70 settlements as observations of the changing size of the fractal city, not scale changes derived by aggregating curves for individual settlements.

However, the dimensions reported here are likely to have the same order of magnitude as those we will compute in the next sections for the envelope–area and envelope–field relations, and these, as we argued earlier, will be less than those which we will compute from the population–area and population–field relations. This is a consequence of the different ways in which the urban boundary is represented as an envelope rather than a perimeter, and strikes at the heart of the argument as to which 'development' should be included in analyses of urban density. The urban envelopes which make up the OPCS data base each include urban areas which nevertheless have zero population density through space occupied by industrial, commercial or educational land uses, by transport infrastructure or by public open space. By contrast, fine resolution raster representations of urban areas maintain 'holes' of unoccupied land within the outermost urban boundary. This explains why analysis of vectorized urban envelopes yields lower fractal dimensions, although the measurements will remain internally consistent between settlements. Moreover, when we examine the distribution of the individual fractal dimensions computed here, there is no real evidence of any spatial patterning, suggesting that boundary geometry alone is not a sufficiently strong criterion to enable classification of urban form.

10.5 Estimates of Allometric and Fractal Dimension in Norfolk

Central to the assessment of urban shape and form is the notion that the growth of urban areas is fuelled by the functions that each area performs in relation to the rest of the urban system. As we noted earlier, established thinking on the nature of urban densities has paid scant attention either to the juxtapositioning of settlements or to the relationship between population growth and boundary shape. However, the development of analogies between growth through diffusion-limited aggregation (DLA) and processes of urban development offers some prospect for understanding how urban forms and densities evolve within a clearly-specified pattern, whilst investigation of envelope–area relations may reveal how growth occurs at the margins of settlements. Thus both may be seen to complement those more established allometric approaches which reduce form to a simple area measure; hence our approach may contribute towards a more sensitive and comprehensive treatment of urban population size and form.

Our present empirical analysis is restricted in the degree to which the artifacts of urban growth can be clearly identified. We have already defined the set of urban area data $\{A_k\}$ through the digitized envelope data $\{E_k\}$ in the OPCS urban areas data set, and population $\{N_k\}$ is also a part of this data set. However, with respect to our DLA analogies, we do not have data on the field area U_k or the radius R_k ($= \sqrt{U}$). In the absence of information as to where the historical 'seed' of each settlement is likely to lie, we can calculate a crude approximation to its radius, using Feret's diameter (F_k) shown in Figure 10.1(d) for Norwich; this enables us to devise a rudimentary 'field' for each of the settlements, and a 'radius' R_k which is taken as $F_k/2$. A further problem is that the rate of urban growth is likely to be uneven at different places around our envelopes, and it remains to be seen whether any signals attributable to characteristic growth patterns might be detectable from aggregate measures of the structure and character of the entire set of boundaries. To provide some indication of the way the urban area data set $\{A_k\}$ relates to the calculated field areas $\{U_k\}$, Figure 10.5 illustrates that the relationship between built-up area and field across the range of settlement sizes is quite erratic, although there is a low positive correlation as might be expected. What Figure 10.5 does show, however, is that urban fields are everywhere much larger than urban areas, thus indicating that none of the settlements in the data set is compact, and that all must be irregular, possibly dendritic, and thus fractal in some sense.

In our empirical analysis of the Norfolk data set, we will examine the four sets of relations identified previously. These are: the population–urban area relation based on equation (10.9) in accordance with established allometric analysis; the population–radius relation based on equation (10.10) in analogy with urban forms generated by DLA; the envelope–area relation based on equation (10.11) which enables us to identify whether there is any detectable evidence that boundaries are characteristic of growth processes; and the envelope–radius relation based on equation (10.12) to identify whether the boundaries of the settlements can be related to fractal growth. Figure 10.6 illustrates each of these relations for the 70 settlements based on logarithmic transforms of the data as implied by equations (10.13) to (10.16), and we have fitted regression lines to the scatters shown in Figure 10.6. The results are shown in Table 10.1.

These results generally confirm our *a priori* expectations. The dimension Δ of the allometric population–urban area relationship is 2.085, close enough to our hypothesized value of 2 to suggest that density is more or less constant with settlement size. Our analysis was carried out for a smaller range of settlement size than previous analyses, and the implication of this

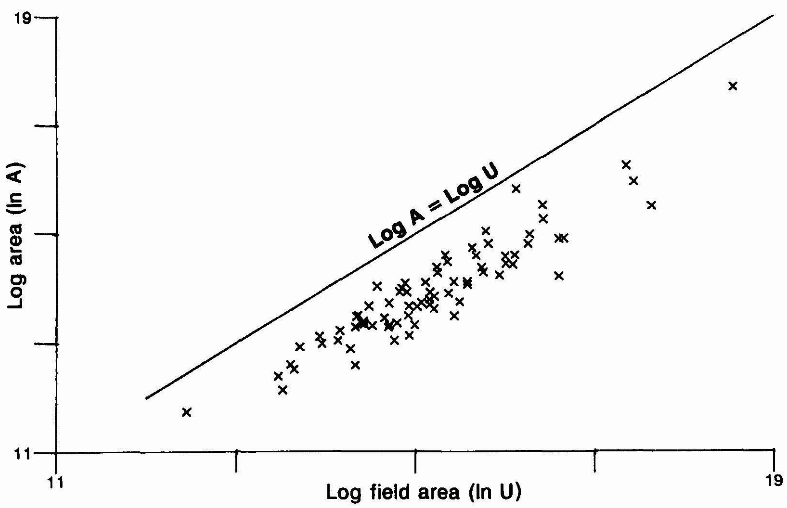

Figure 10.5. The relation between urban area and urban field.

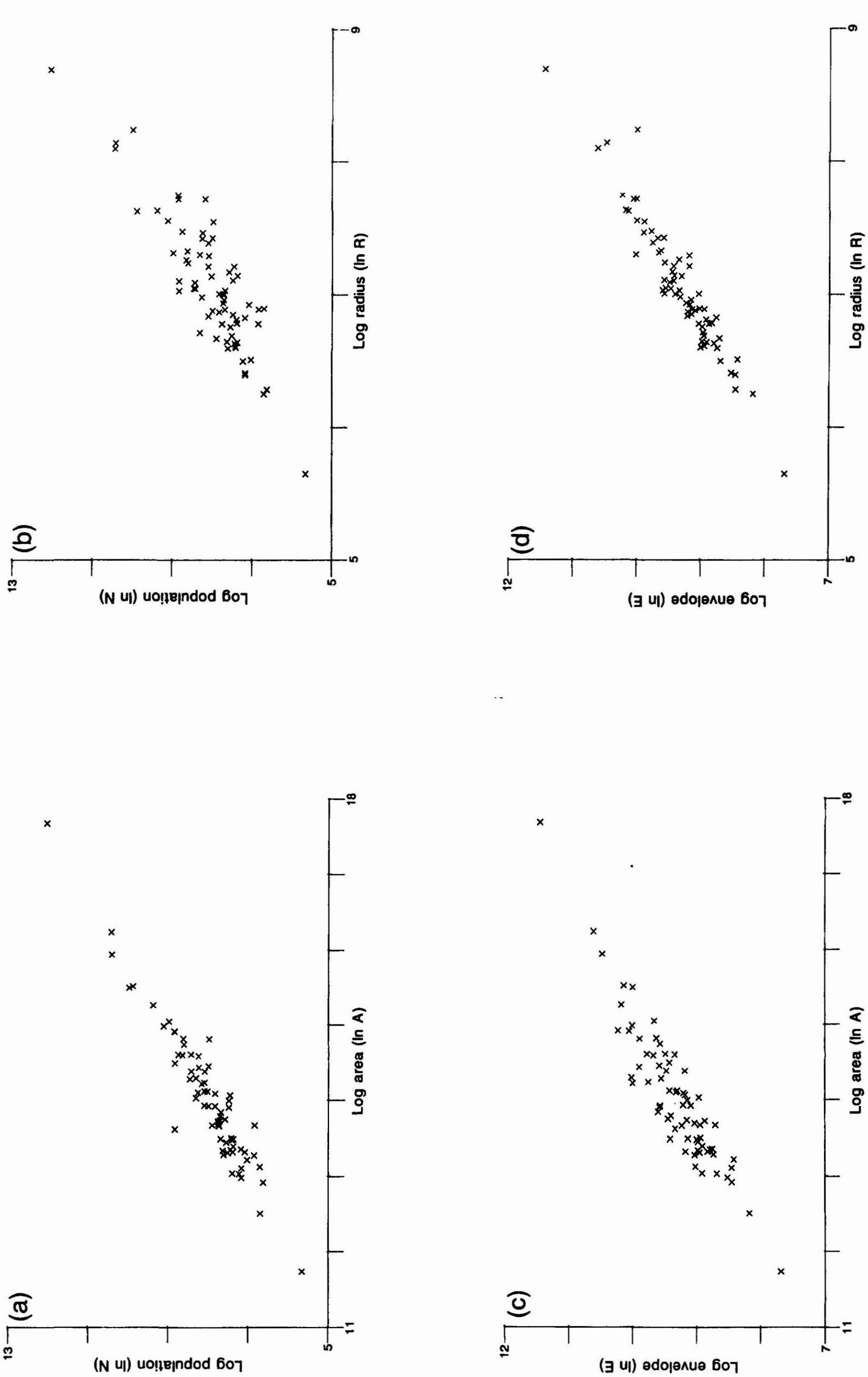

Figure 10.6. Allometric and DLA relations for the 70 urban settlements.

Table 10.1. Estimated dimensions for 70 urban settlements

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
Slope coefficient	1.043	1.738	0.613	1.152
r^2	90.3	76.1	85.7	91.5
Dimension	2.085	1.738	1.227	1.152

Note: in this and subsequent tables in this chapter, the r^2 statistic is the coefficient of determination which gives the percentage of the covariation explained by the relationship.

finding is to reinforce the simple scaling hypothesis based on an area–area relation found by Woldenberg (1973) and Dutton (1973), rather than the area–volume hypothesis argued by Nordbeck (1971). The r^2 statistic suggests a high global goodness-of-fit, although the high degree of potential leverage exerted by the three largest settlements is a potential source of uncertainty. The dimension estimated from the population–radius analysis is very close to that of a classic DLA structure with $D = 1.738$, and this is an encouraging result, particularly in view of the crudity of the approximation to settlement radius. However, the level of overall statistical fit is lower, with only 76% of the variance explained, and high potential leverage effects can again be detected from Figure 10.6(b). Both of the envelope analyses produced high fitting estimates of their dimensions with $\delta = 1.227$ and $\check{D} = 1.152$. It is interesting to note that the average dimension of the individual settlement dimensions computed by applying Richardson's (1961) method to the envelopes of each settlement discussed earlier, was 1.148, and this compares quite favorably with the value of $\check{D}$ which is its closest comparator.

Although these results are most encouraging, confirming our initial hypotheses and demonstrating, at least to us, the value of prior theoretical analysis in establishing such hypotheses, we are also concerned to identify whether or not our results can be disaggregated and generalized to subsets of settlements of different sizes and in different locations. Accordingly, we carried out two further sets of analyses on the data. First, the two largest outlying settlements representing Norwich and King's Lynn in the graphs of Figure 10.6 were removed from the data set, first individually and then together. In a statistical sense, this was carried out in order to verify that the high potential leverage effect of these observations was not exerted too strongly against the dominant trend in the data points. In a theoretical sense, this was also important in so far as all of the size and area relations confirm that these two settlements are the most important in the study area, and thus that they might exhibit different relations between density and form. The results of this analysis are shown in Table 10.2(a)–(c). The r^2 statistics shown there are consistently lower than the corresponding values in Table 10.1, indicating that the major settlements accord with the general trend in the rest of the data. With the exception of the envelope–urban area relation, all of the analyses which exclude Norwich and/or King's Lynn

Table 10.2. Estimated dimensions of the urban settlements excluding the largest towns

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(a) Excluding Norwich				
Slope coefficient	1.024	1.603	0.624	1.125
r^2	87.4	71.7	82.6	89.9
Dimension	2.048	1.603	1.247	1.125
(b) Excluding King's Lynn				
Slope coefficient	1.038	1.698	0.616	1.146
r^2	89.4	74.5	84.7	90.9
Dimension	2.075	1.698	1.233	1.146
(c) Excluding Norwich and King's Lynn				
Slope coefficient	1.014	1.541	0.629	1.115
r^2	85.7	69.3	81.0	89.1
Dimension	2.029	1.541	1.259	1.115

produce lower fractal dimensions, suggesting that the global figure is boosted by the particularly tentacular structure of these two settlements.

The second set of disaggregate analyses considered the relations within several subsets of settlements defined *a priori*. Three classes were identified: two regions were delineated around the hinterlands of Norwich and King's Lynn, whilst a third was drawn to embrace all of the settlements along the coast. Settlements which did not clearly fall into any of these categories were omitted. This regionalization is shown in Figure 10.7. The rationale for the first two functional regionalizations was twofold: first, to identify whether the settlements *within* two more broadly-defined urban fields, approximating the sphere of influence of each of the two largest settlements, shared common characteristics; and, second, to make a first attempt at identifying common characteristics between them. The results shown in Table 10.3(a)–(c) suggest that although the Norwich region appears to generate higher dimensions than the King's Lynn area and the full set of 70 settlements (Table 10.1), no startling differences emerge.

The rationale for separating out the coastal region was to identify how the constraining impact of the sea restricts the shape and form of the settlements. All of the four dimensions – Δ, D, δ and $\check{D}$ – will fall in value if the space within which any settlement can grow is restricted. This is an obvious consequence of constraining the geometry and this effect has been clearly demonstrated by the simulated urban growth patterns using DLA presented in Chapter 8. In fact, this effect can be seen in Table 10.3 for the

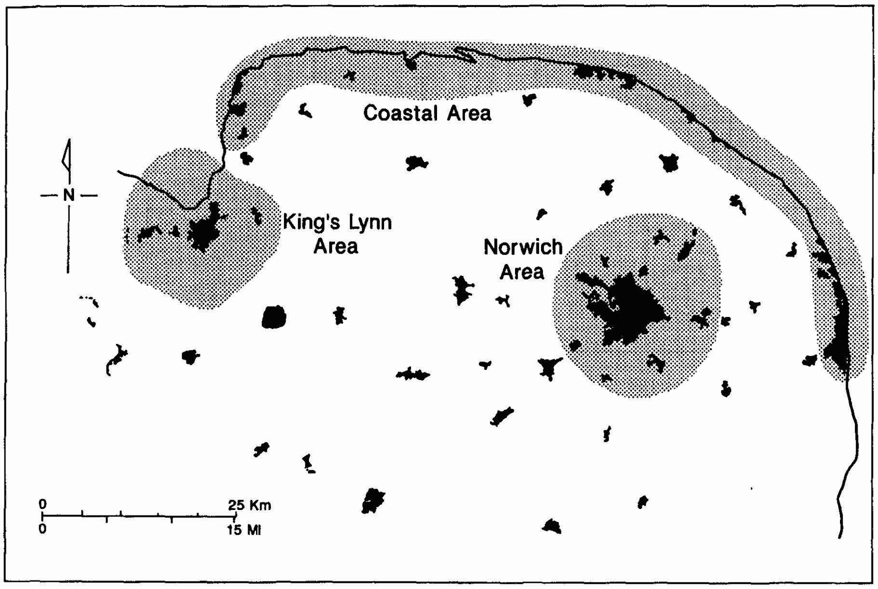

Figure 10.7. Regionalization of the Norfolk settlement pattern.

Table 10.3. Estimated dimensions for three regionalizations of the urban settlement pattern

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(a) Norwich region				
Slope coefficient	1.040	1.980	0.601	1.300
r^2	96.3	83.9	86.5	97.2
Dimension	2.080	1.980	1.202	1.300
(b) King's Lynn region				
Slope coefficient	1.010	1.750	0.623	1.260
r^2	94.0	74.4	90.4	97.6
Dimension	2.020	1.750	1.246	1.260
(c) Coastal region				
Slope coefficient	1.010	1.630	0.634	1.030
r^2	75.4	72.3	90.8	87.9
Dimension	2.020	1.630	1.268	1.030

DLA dimension associated with the form of the Norfolk coastal settlements. The slightly higher dimension of the envelope–area relation reflects increased concentration of growth upon the inland portion of each of the settlements, although the dimension of the envelope–radius relation is lower, reflecting the restrictions upon the growth field. From Tables 10.2 and 10.3, it is also significant that it is the DLA dimension D which shows the greatest sensitivity to our regionalization varying from 1.603 to 1.980, in contrast to the other three dimensions where the range of variation is much narrower.

10.6 Constraining Urban Form Through Green Belts

A major problem which we have in one sense avoided, apart from in our theoretical simulations in Chapter 8, concerns the effect of geometric constraints on the city system; these involve the extent to which space is filled, the density of development, and the parameter values of the scaling relations. Clearly, *ceteris paribus*, the less space available, the lower the fractal dimension, and this is especially clear when we consider cities that develop in coastal regions or in areas where a major part of their hinterland or field is constrained from development. However, notwithstanding the problems of assessing these effects, we can turn these problems to our advantage in exploring the impact which known constraints might have had on the development of cities. A particularly important constraint on the form of the city system in Britain has been the impact of planning policies which have sought to constrain and inhibit development around major cities during the last 60 years. The most explicit policy instrument used to effect these policies has been the 'Green Belt', and using our scaling analysis, we will now attempt to measure this impact.

The idea of a 'Green Belt' of open land encircling a major city and embracing both small and medium-sized settlements located in the hinterland of a 'core' city is one of the main philosophical and practical underpinnings of the British Town and Country Planning system (Ravetz, 1980). As such, both the idea and the practice of Green Belts as a planning policy instrument have been debated and implemented most extensively in relation to the Metropolitan Green Belt (MGB), an annular tract of land now extending for between 25–40 km in width around the Greater London conurbation. Not unnaturally, given both its scale and importance and the nature of the development pressures upon it, the MGB has, over the years, been the subject of considerable research. Attention has been focussed on such matters as the distribution of land uses within it, its impact on land prices within urban areas, the function it performs in terms of human activities and who gains and who loses from its continued existence (Hall *et al.*, 1973; Munton, 1983; Elson, 1986; Evans, 1989).

Although the origins of the MGB (and indeed of Green Belts generally) can be traced to the Garden City Movement pioneered by Ebenezer Howard (1898, 1965) and the more conceptually based work of Raymond Unwin for the Greater London Regional Planning Committee (1927–36), the main postwar impetus for the implementation of a complete *cordon sanitaire* around London came from Sir Patrick Abercrombie's 1944 Greater London Plan (Abercrombie, 1945). This had the multiple aims of stopping the outward growth of London itself, preserving open land for agriculture and recreation and preventing the coalescence of towns contained within it. In 1946 the multiplicity of aims contained within Abercrombie's Green Belt proposals were accepted by central government, although the over-riding objective was, and continues to be, to contain the growth of urban areas (Elson, 1986). The broader policy was to be effected through the development plan provisions of the Town and Country Planning Act 1947, and its proposals were implemented with a certain enthusiasm by the seven county planning authorities surrounding London (Mandelker, 1962). The present extent of the MGB was basically established in the Structure Plans of the mid-1970s (SERPLAN, 1976) and although there were four main categories of Green Belt in operational terms (i.e. originally submitted and approved, approved extensions, extensions with interim approval and areas where Green Belt controls were operated with central government acceptance), to all intents and purposes broadly similar restraint measures became operative over the whole MGB area (Elson, 1986).

The context and the means for containing growth was set out in two circulars issued by the Ministry of Housing and Local Government in 1955 and 1957. The first established the objectives of Green Belt controls. These were: to check the further growth of a large built-up area; to prevent neighboring towns from merging into one another; and to preserve the special character of a town (MHLG, 1955). From this point on, therefore, the statutory support for operating development controls within Green Belts rested ultimately on concerns about urban form (and, working indirectly through form on urban functions) and not on the preservation of urban land for agriculture or recreation (Elson, 1986). The second circular (MHLG, 1957) introduced, among other things, the concept of 'white land' parcels between the town and the Green Belt which would not be developed in the contemporary plan period but which could be developed later without prejudice to the strategic and local objectives of a Green Belt. Thus whilst the objective of Green Belt planning was to be the control of urban form, there was also scope for some locally declared policy which might, in the longer term, result in a changed settlement pattern (Elson, 1986).

We might anticipate that this dual strategy of central direction about aims and local autonomy about means has had an impact upon the nature and form of settlements, yet this is a subject which has never yet been researched in anything but superficially descriptive terms. The only extensive studies of the impact of Green Belts upon urban form are those carried out by Elson and his colleagues, and these show that, for a very small number of settlements, the provision of 'white land' on the periphery of settlements was indeed a significant local determinant of change in the pattern of urban land uses (Elson, 1986). Clearly, however, there is a need for a more broadly based and systematic empirical analysis of the impact of physical planning controls such as Green Belts on the form of urban settlements.

In this chapter, we will make a first attempt to address this issue, using the Office of Population Censuses and Surveys (OPCS) urban areas data-base (OPCS, 1984) which we used for the Norfolk example. Using again the four scaling relations based on the allometric and fractal growth of a system of cities and given earlier in equations (10.9) to (10.12), we will consider the degree to which the form and density of urban settlements has been influenced by Green Belt designation, and we will attempt to discern apparent variations in the spatial manifestations of what is first and foremost a national development policy. To this end, we will develop a straightforward analysis of the physical extent of urban areas in South East England and attempt to interpret shapes and forms with respect to the presence or absence of direct Green Belt Policy on their development. We will also draw some general conclusions as to the prospects for devising more coherent settlement classification systems which incorporate quantitative measures of shape, dimension and density. Our analysis thus seeks to link our new measures of urban shape and form to the practical consequences of policies which seek to mold and constrain urban development. It is in this sense that our analysis is preliminary, and thus represents only a starting point for a broader research agenda.

In defining the impact of physical planning policies, particularly those involving restricting urban development using instruments such as Green Belts, it is essential to evaluate their effects by examining the extent to which the physical form of development departs from the 'norm'. In this quest, we need to define urban form not only in terms of the size of development but also in terms of its shape. This is important because policy instruments such as New Towns and Green Belts have often been implemented in terms of idealized forms such as those characterizing the British New Towns and Garden Cities. But a rigorous study of the size and shape of urban settlements, however, is in its infancy. Despite the emphasis in land use planning upon controlling and influencing the size and shape of towns, most work has hitherto been cast in a somewhat idealistic mold, reflecting a fascination with form and shape for its own sake rather than as a consequence of the processes and decisions which condition the spread of urban settlement.

We are now in a position to make clear our strategy for the analysis of the impacts of Green Belts on urban form using scaling relations. In essence, what we will do is compute these measures for different classes of settlement, each of which is classified according to the policy instruments which have been applied in the control of their development. As we do not have parameter values of the four relationships for a given baseline, we will also be concerned with estimating the parameters of this baseline. In short, we need to develop the following estimates of values associated with the entire data set of settlements, the values associated with those settlements which are unlikely to have been affected by Green Belt Policy, and then those that have been so affected. It is thus the differences in parameter values between these various sets that we will be focussing upon. Before we present these, it is worth noting that the analysis could be inconclusive if our estimates of the values associated with the control baseline – the set of settlements not affected by policy instruments – are not significant or imply a poor performance of the model relationships. The same might be true of other sets of estimates, and thus there is always the possibility that our assessment of impact will be dwarfed by poor performance or contradictory results from the various estimations.

Before we develop the analysis, we must explain briefly the data set which we will present in the same way as we did the Norfolk data. A subset of the OPCS data base pertaining to all of the urban areas in the South East England planning region is shown in Figure 10.8, and it is useful to compare this to Bracken's (1993) visualization illustrated in Plate 7.1. Although the largest urban areas (notably London) are broken down into boroughs and districts in the original data set, these administrative divisions have been removed for purposes of our analysis. What remains for these largest settlements is a number of large polygons which describe the bounding envelopes of contiguous urban development. We recognized at the outset of our analysis that our posited relationships between settlement populations and the shapes of urban areas are unlikely to hold over the entire range of settlement sizes. Specifically, the geometry of those smallest settlements which comprise a mere handful of inhabited buildings are likely to be dominated by the intersection of transport links, and thus will reflect the nature of the local and regional transport network rather than the intrinsic characteristics of growing settlements *per se*. As previously, the smallest settlements in the data base were thus deemed irrelevant in terms of both population size and areal extent, and thus removed.

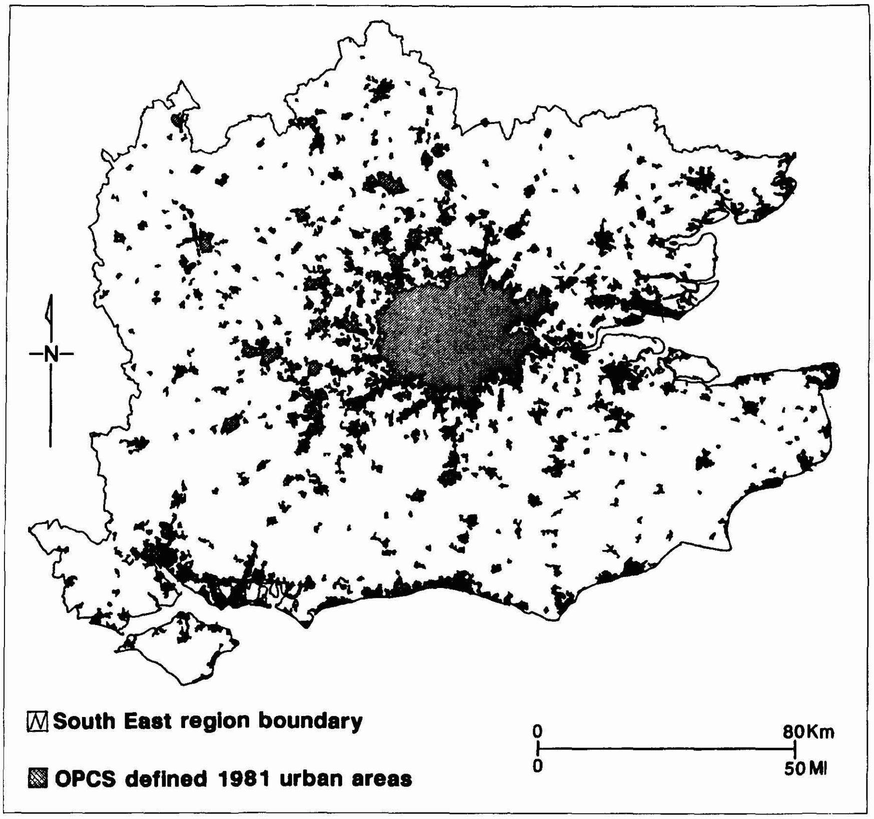

Figure 10.8. Urban areas in South East England.

Figure 10.9 illustrates this for the relationship between population and area in the whole digitized settlement system of the South East. There is a reasonably clear break in the dominant relationship amongst the very smallest settlements, and although we have estimated empirical relationships using the entire data set (Table 10.4), these results are neither statistically efficient nor theoretically coherent. In the bulk of our analysis, we have adopted the practice of excluding all settlements whose form was encoded using 15 or fewer digitized points, since such settlements were deemed too small for our specific purposes. This amounts to a fairly minor amendment of the Department of the Environment definition of 'urban' land use and reduced our data set from the original 701 observations to 686 settlements.

Since the historical center point of each urban area is not digitized as part of the data set, we have approximated the settlement radius as being equal to half of the spanning distance joining the two widest spaced digitized points on the settlement boundary (that is, half the Feret diameter). A further complication in the data set is that the population figures for the urban areas are not assigned to all of the individual parcels which together comprise a single named settlement. This means that exact population figures cannot be attributed to approximately 60 settlements. In practice, this was resolved by allocating population to physically split named settlements in direct proportion to the area of the constituent parcels. This does not affect the weighting of such parcels in our regressions, although if such named settlements are outliers to the main scatter of points, this does result in the appearance of a parallel scatter of points about the main trend in the data, as is clearly seen in Figure 10.9.

10.7 The Impact of Green Belts Using Scaling Relations

Figure 10.10(a)–(d) illustrates each of the logarithmically transformed scaling relations given in equations (10.13) to (10.16) for the 686 settlements

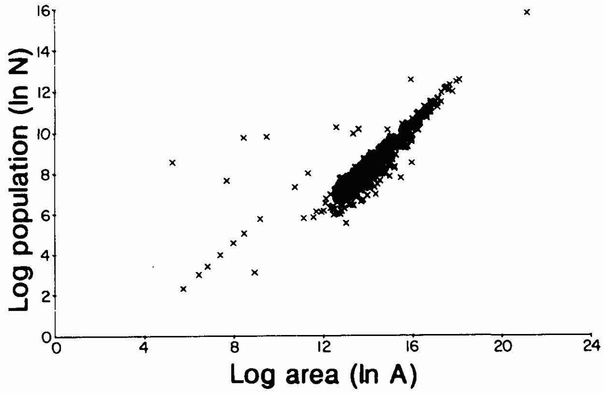

Figure 10.9. The population–area relation for the entire South East England settlement system.

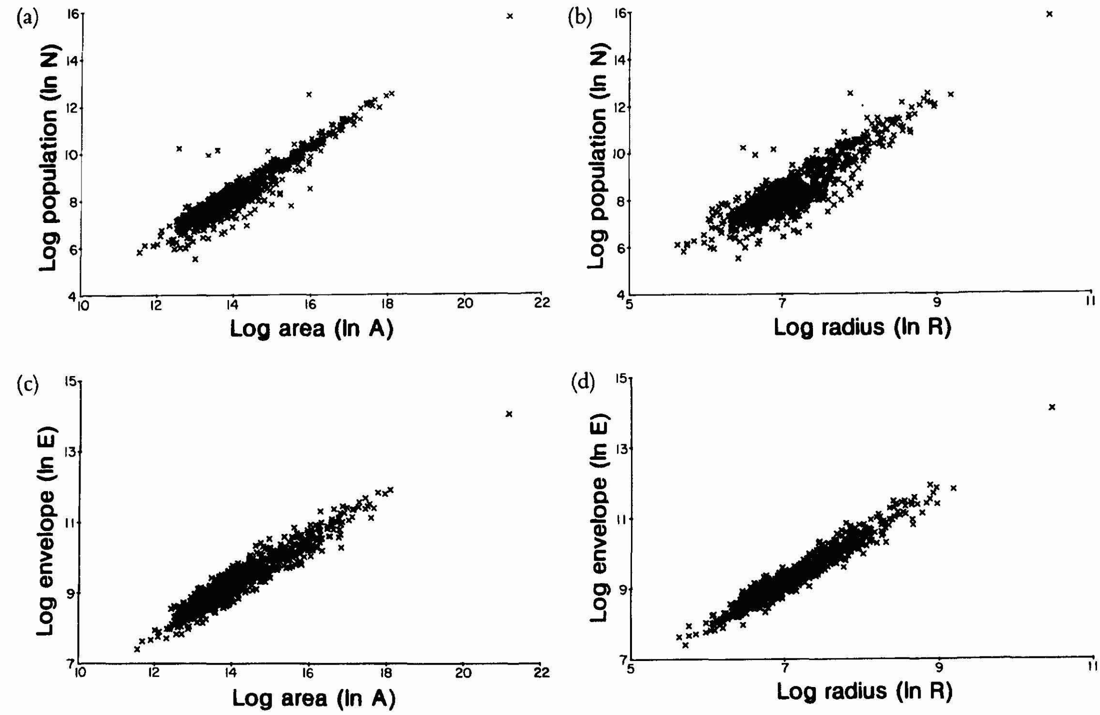

Figure 10.10. Allometric and DLA relations for the usable settlement system.

which were captured with 16 or more coordinate pairs. The results of fitting regressions to the scatters shown in Figure 10.10 are given in Table 10.4, and the 95% confidence intervals about the dimensional estimates are reproduced in diagrammatic form in Figure 10.11(a)–(d). In interpreting these results, we will also draw comparisons with our previous empirical study of the settlement structure of Norfolk. It was recognized at the outset that no region of England even approximates the isotropic surface on which, for example, central place theory is developed, although Norfolk was chosen for our first analysis because of the comparative homogeneity of its terrain and the absence of abnormal planning restrictions upon urban growth.

The results of our analyses of the South East England data generally conform to our *a priori* expectations. There are evident differences in the parameter and dimensional estimates between the analyses embracing all (701) settlements and those (686) settlements comprising 16 or more coordinate pairs. In the cases of the population–area, population–radius and envelope–area relations, these differences are statistically significant. The classic population–area relationship has dimension 2.046, which is quite close to (although, at conventional confidence levels, just above) the widely mooted value of 2. We made a similar finding in our Norfolk study, where a similar degree of overall statistical fit (r^2, corrected for degrees of freedom) was discerned. This general consistency between study areas is encouraging, particularly in view of the inclusion of London as an observation. London clearly constitutes a high potential leverage point in the analysis, although it is theoretically suspect to exclude the observation purely on grounds of

Table 10.4. Estimated dimensions for settlements in the South East region[1]

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(a) All settlements (701)				
Slope coefficient	0.808	1.569	0.619	1.258
r^2	75.6	70.8	93.1	95.6
Dimension	1.616	1.569	1.238	1.258
(b) All usable settlements (686)				
Slope coefficient	1.023	1.872	0.645	1.271
r^2	89.0	79.0	91.1	93.9
Dimension	2.046	1.872	1.290	1.271
(c) Outside the Green Belt (389)				
Slope coefficient	1.047	1.868	0.635	1.236
r^2	89.1	77.6	89.8	93.0
Dimension	2.093	1.868	1.271	1.236
(d) Partly in the Green Belt (15)				
Slope coefficient	1.083	1.890	0.719	1.317
r^2	94.1	84.3	92.9	92.4
Dimension	2.167	1.890	1.439	1.317
(e) Within the Green Belt (237)				
Slope coefficient	0.994	1.875	0.663	1.323
r^2	93.7	86.0	93.3	95.8
Dimension	1.988	1.875	1.326	1.323

[1]In Tables 10.4 to 10.6, data pertaining to settlements that lie within or astride the Oxford and the Southampton Green Belt boundaries have been omitted from the analyses. The number in parentheses after the analysis label is the number of settlements in that category.

size, since the impact of the Green Belt is likely to be most significant along and around the boundary of this area. In practice, however, this potential leverage transpires not to be against the trend in the rest of the data, and an exploratory analysis carried out with this dominant central settlement excluded, yielded results which were neither more consistent in substantive terms nor were significantly improved in terms of statistical fit.

The result of the population–radius regression yields a significantly higher dimension than was anticipated on *a priori* grounds, suggesting that settlements in the South East fill more of their urban fields than does the classic space-filling diffusion-limited aggregation model. This was not the case in any of our previous studies in which the DLA structure provided

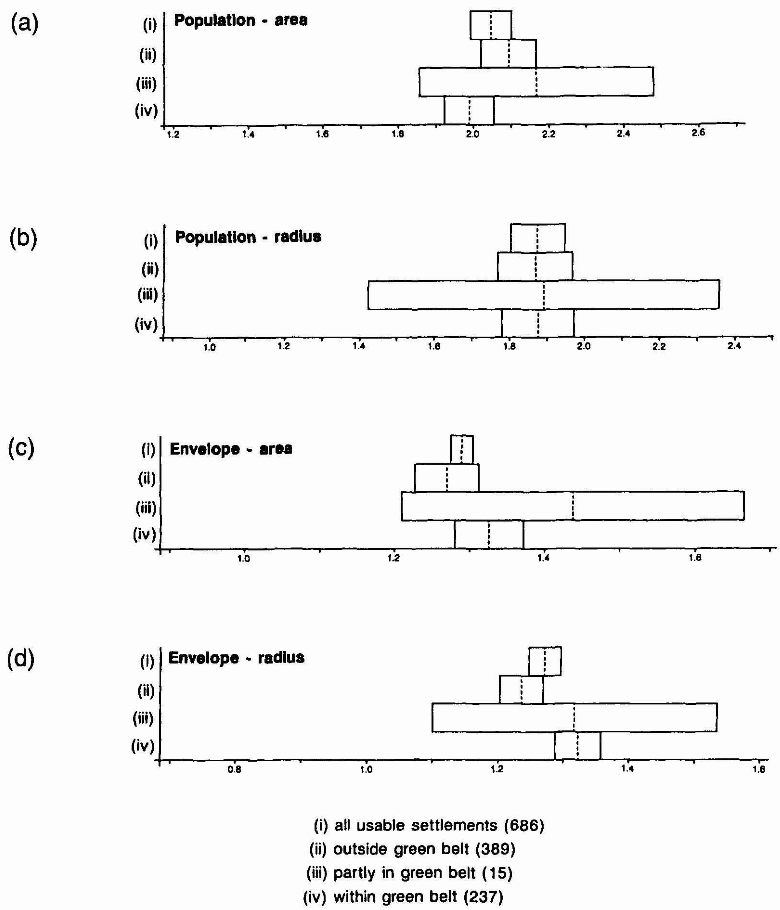

Figure 10.11. Confidence intervals about the dimension estimates (a) population–area; (b) population–radius; (c) envelope–area; (d) envelope–radius.

a plausible theoretical baseline model, and may be taken to imply that pressures conspire to encourage the development of more intricate settlement forms within the urban fields of settlements in this region. The purely geometrical analyses yield values consistent with our expectations, and high levels of statistical fit characterize these relationships.

As the next step, the South East settlements were divided into three groups according to their position relative to the Greater London Green Belt: those (237) settlements which lay entirely within it; those (389) that lay entirely outside of it; and those (15) that lay astride the boundary. The South East region includes two other Green Belts, centered upon Oxford and Southampton. For purposes of our present analyses, it was considered that these Green Belts were different in spatial and temporal terms from the London Green Belt, and thus settlements that lay either within or astride the Oxford and Southampton Green Belt boundaries were omitted from our analysis at this stage. This classification is shown in Figure 10.12. The results of separate regression analyses upon these subareas are shown in

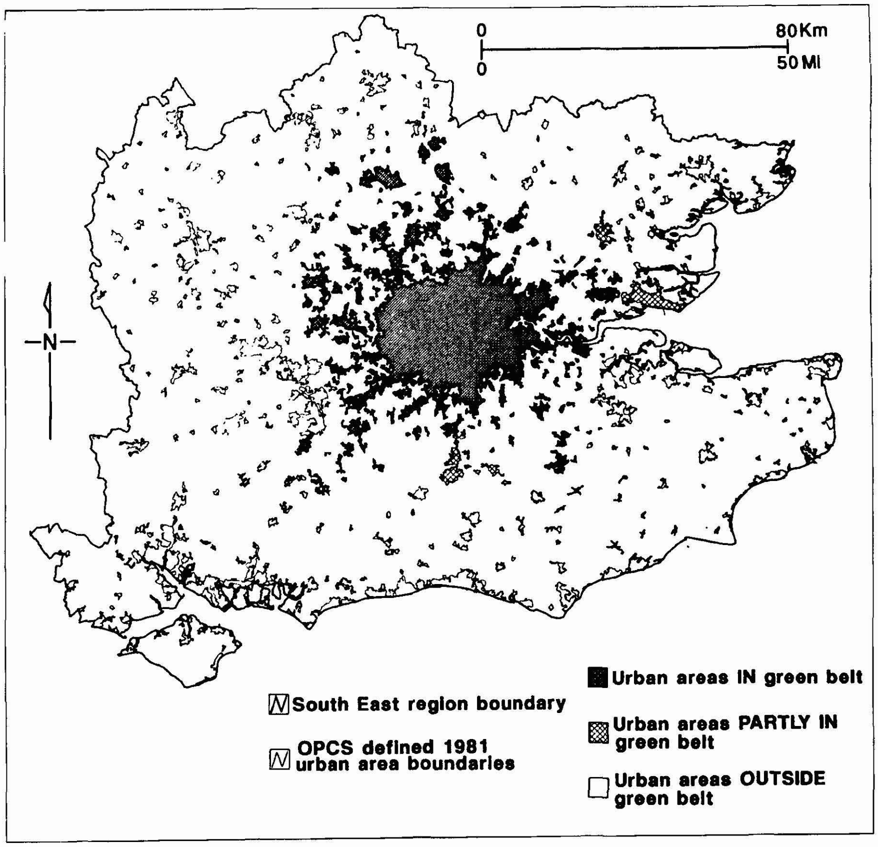

Figure 10.12. The Green Belt status of settlements in South East England.

Table 10.4 and Figure 10.11. There is no significant difference between the estimated dimensions for the population–area relation, although the wider confidence intervals and the lower r^2 value for the extra-Green Belt settlements are indicative of greater variation in the effects of forces governing this relation. The lower estimated dimension for the population–area relation for the intra-Green Belt settlements is indicative of a disproportionately small increase in area amongst larger Green Belt settlements, although the global level of statistical fit is insufficient to confirm an unequivocal difference.

Neither are clear distinctions apparent when considering the population–radius relationship. Here, the estimated dimension and the extent of the confidence limits are remarkably similar for all of the settlement classes, although the modified r^2 statistic suggests greater heterogeneity amongst the extra-Green Belt settlements. Regarding the envelope–area relation, there is very limited evidence to suggest that the larger settlements which straddle the Green Belt boundary exhibit disproportionate increases in boundary length, and this might be indicative of contortions in urban form consequent upon differential planning restrictions. However, largely because of the small number of observations, no statistically significant differences are apparent. Significant differences do, however, exist, between the envelope–radius relations for extra- versus intra-Green Belt settlements.

The intra-Green Belt dimensional estimate is higher, suggesting that these settlements are more circular and compact than those outside.

These, then, are our preliminary attempts to utilize detailed vectorized boundary data in order to gauge the general spatial impact of an important component of spatial policy. Of course, this discussion presumes that settlement shapes in South East England would be free to evolve in an unconstrained manner in the absence of Green Belt planning policy. The spirit of our approach is to assume that the multitude of other factors which conspire to mold urban form (terrain, fluvial features, land ownership patterns, etc.) do not obscure the central impact of this strict planning control. One of the most obvious and important confounding influences is that of the coast, which clearly has constrained the shape, form and density of many settlements in our study region. Consequently, a separate set of analyses were carried out in which the coastal settlements illustrated in Figure 10.13 were excluded. The results are presented in Table 10.5, and show that there exist some minor differences in dimensional estimates and confidence intervals and that the previously significant difference between the 'partly in' and 'outside' dimensional estimates for the envelope–area relation disappears. The results nevertheless show the same broad relationships as identified in Table 10.4, and the maintenance of the population–radius differ-

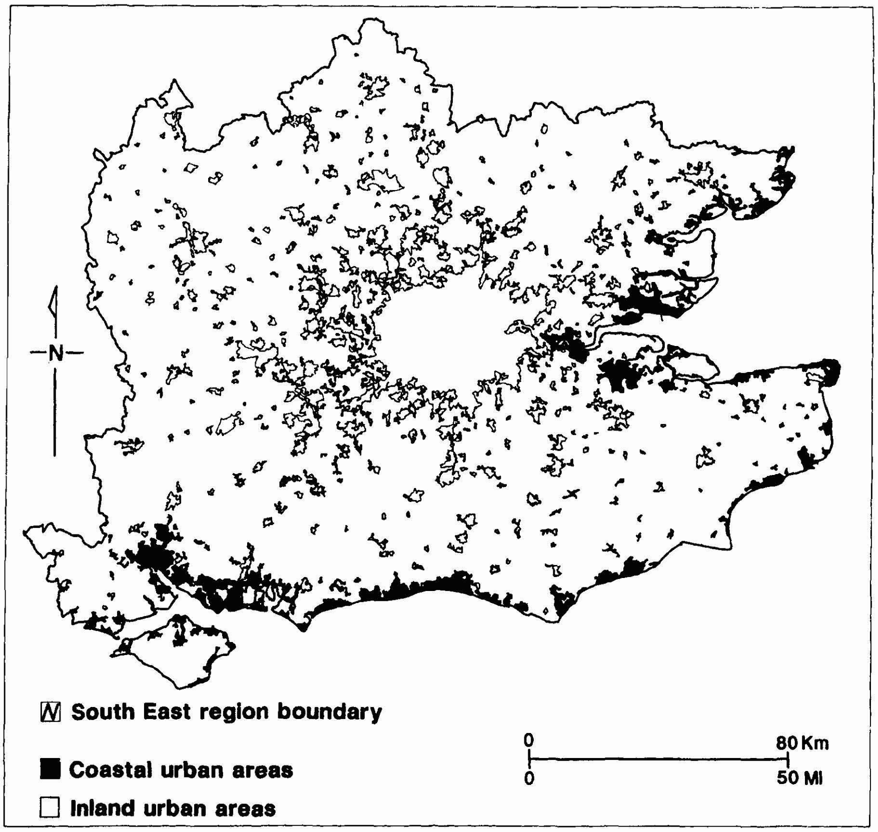

Figure 10.13. Coastal settlements excluded from the analysis.

Table 10.5. Estimated dimensions for settlements in the South East region excluding coastal settlements

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(a) All usable settlements (592)				
Slope coefficient	0.998	1.836	0.645	1.289
r^2	88.0	77.6	90.3	93.9
Dimension	1.996	1.836	1.290	1.289
(b) Outside the Green Belt				
Slope coefficient	0.996	1.744	0.628	1.250
r^2	84.4	69.8	86.1	92.3
Dimension	1.992	1.744	1.256	1.250
(c) Partly in the Green Belt (8)				
Slope coefficient	1.005	2.270	0.650	1.486
r^2	96.5	93.9	92.6	92.6
Dimension	2.010	2.270	1.300	1.486
(d) Within the Green Belt (222)				
Slope coefficient	0.997	1.903	0.661	1.323
r^2	94.2	87.2	94.0	95.8
Dimension	1.995	1.903	1.321	1.323

ences suggests that the distorting impact of the sea is less than that generated by Green Belt planning policy.

In a final series of analyses, we have begun to investigate whether our empirical settlement relations vary between County Planning Authorities. It is conceivable that, over half a century, different County Planning Authorities have evolved consistently different interpretations of Green Belt Policy. Our four relations were thus estimated for each of the 13 county divisions within the South East Region (see Figure 10.14), although the small number of usable observations for a few of these counties leads to quite wide confidence intervals. In the case of Greater London, the four relations were estimated for each of 36 administrative divisions of the area, and so these results are not strictly comparable with those of the other counties. The results of this county-based analysis are reproduced in Table 10.6 and Figure 10.15. There are no evident significant differences amongst the population–area and population–radius results, suggesting that population pressures across different counties have not had the effect of distorting regional population density norms.

However, there is evidence that the settlement geometry differs between

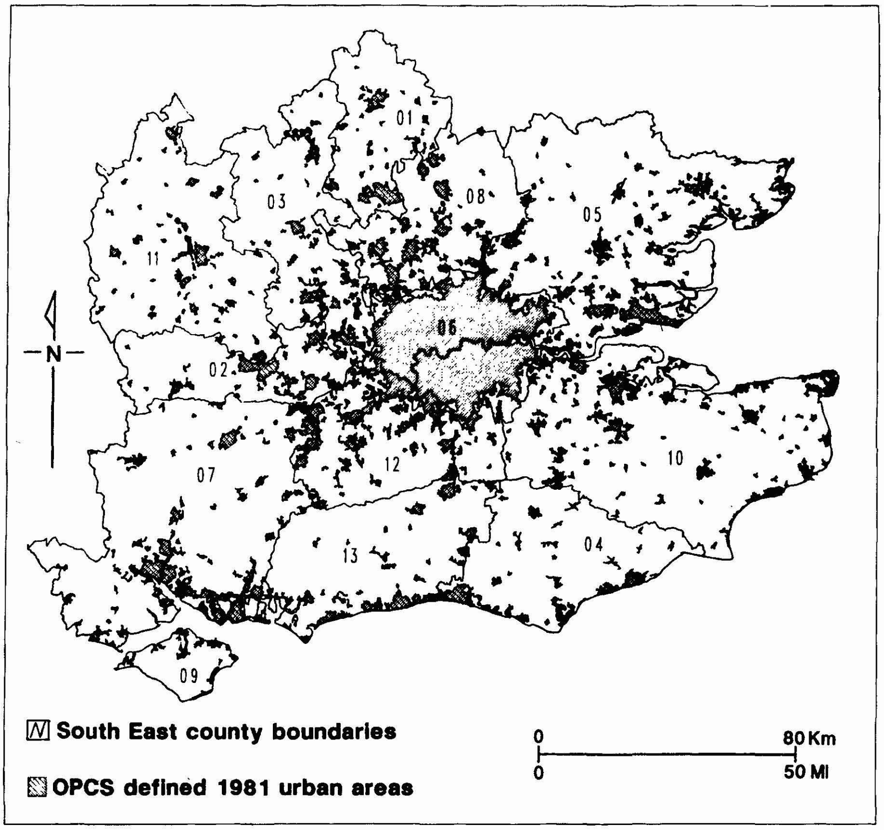

Figure 10.14. County divisions in South East England.

individual counties. First, the envelope–area relation suggests that bounding envelopes are significantly shorter for a given settlement area in Oxfordshire and Hertfordshire than in any of Kent, Surrey, Greater London and (for the case of Oxfordshire only) Berkshire. The envelope–area relations for these two counties are also significantly smaller than the estimates derived from the complete set of (686) settlements (Table 10.4). This can be seen as indicative that growth has been contained within more compact areas in these two counties. The envelope–radius relation for Oxfordshire also exhibits a significantly lower dimensional estimate than for the set of all settlements and than for the individual counties of Berkshire, Essex, Hertfordshire, Kent, Surrey and Greater London, suggesting that growth in Oxfordshire has been contained within more compact areas than has been the case in these other counties. A significant difference in the envelope–area relation also exists between Buckinghamshire and Surrey.

10.8 An Unfinished Agenda

So far we have identified statistical differences between the various subgroupings of settlements based on the implementation of Green Belt

Table 10.6 Estimated dimensions for the County-based settlement analysis

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(a) Bedfordshire (33)				
Slope coefficient	0.854	1.564	0.622	1.208
r^2	64.3	57.7	93.7	95.2
Dimension	1.709	1.564	1.244	1.208
(b) Berkshire (41)				
Slope coefficient	1.027	1.908	0.663	1.272
r^2	95.0	90.3	94.7	96.0
Dimension	2.055	1.908	1.327	1.272
(c) Buckinghamshire (51)				
Slope coefficient	0.995	1.733	0.644	1.205
r^2	96.1	87.4	90.6	95.2
Dimension	1.989	1.733	1.289	1.205
(d) East Sussex (29)				
Slope coefficient	1.150	1.971	0.595	1.187
r^2	94.8	72.8	84.8	89.5
Dimension	2.300	1.971	1.190	1.187
(e) Essex (87)				
Slope coefficient	1.046	1.891	0.648	1.277
r^2	91.2	78.5	91.7	93.9
Dimension	2.092	1.891	1.296	1.277
(f) Greater London (42)				
Slope coefficient	1.011	1.993	0.683	1.376
r^2	98.4	94.7	96.4	96.9
Dimension	2.022	1.993	1.367	1.376
(g) Hampshire (96)				
Slope coefficient	1.072	1.986	0.629	1.269
r^2	84.1	73.5	89.7	93.1
Dimension	2.144	1.986	1.258	1.269
(h) Hertfordshire (40)				
Slope coefficient	1.040	2.192	0.577	1.311
r^2	97.5	87.0	90.6	94.4
Dimension	2.079	2.192	1.153	1.311

Table 10.6. Continued

Statistic	Population–area $\Delta \approx 2$	Population–radius $D \approx 1.7$	Envelope–area $\delta \approx 1.3$	Envelope–radius $\check{D} \approx 1.2$
(i) Isle of Wight (13)				
Slope coefficient	1.097	1.643	0.637	1.053
r^2	90.7	66.5	86.8	79.4
Dimension	2.194	1.643	1.273	1.053
(j) Kent (93)				
Slope coefficient	0.995	1.759	0.694	1.343
r^2	85.4	72.5	90.5	92.4
Dimension	1.990	1.759	1.388	1.343
(k) Oxfordshire (56)				
Slope coefficient	0.931	1.748	0.530	1.089
r^2	78.1	69.1	84.8	90.3
Dimension	1.861	1.748	1.060	1.089
(l) Surrey (40)				
Slope coefficient	0.965	1.756	0.705	1.389
r^2	93.7	81.7	93.1	95.5
Dimension	1.929	1.756	1.410	1.389
(m) West Sussex (46)				
Slope coefficient	1.124	1.914	0.644	1.202
r^2	90.5	77.7	90.2	93.5
Dimension	2.248	1.914	1.287	1.202

policies, geometrical constraints such as those posed by the coastline, and administrative differences in the operation of planning policies, but we have not commented on the substantive differences which our analysis has revealed. In *a priori* terms, we might expect that where Green Belt Policy is rigidly enforced, this would constrain the form of settlement and development, and in turn would make the boundaries of such settlement more irregular in contrast to development not so constrained. However, such constraints also imply that the amount of space in the field about such settlements would be reduced by Green Belt Policy. This implies that the value of D associated with the population–field relation would be less than that for the unconstrained growth, while the value of $\check{D}$ for the constrained case would be greater than for the unconstrained case. In fact, these hypothesized values are borne out in Table 10.4, although the variance in the parameters of the Green Belt affected settlements is much greater than the unconstrained set of settlements. In the case of the population–area relation,

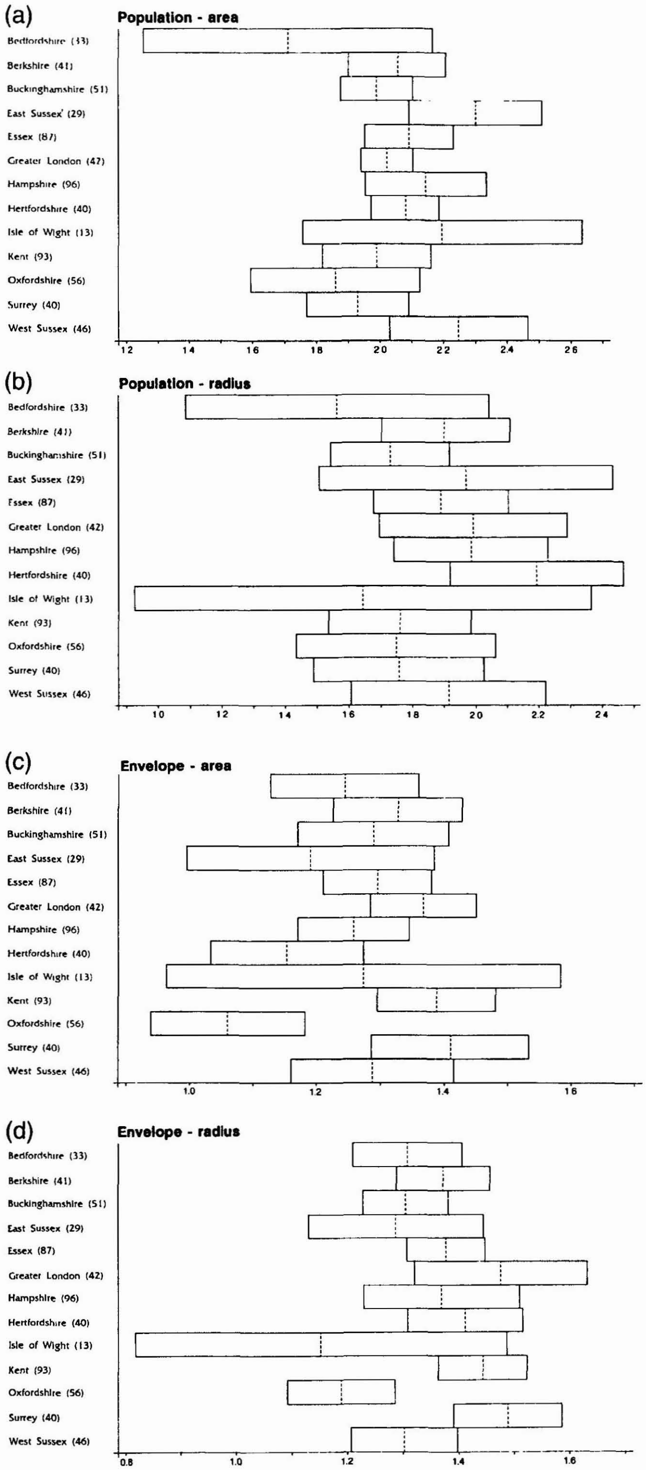

Figure 10.15. Confidence intervals for the county-based dimension estimates (a) population–area; (b) population–radius; (c) envelope–area; (d) envelope–radius.

the parameter of the constrained case is just less than 2, while for the unconstrained it is a little greater than 2. The same degree of difference is borne out in the envelope–area parameter values. In the case of the county-based analysis, there is very wide variation between the purely geometrical dimensions, whereas population-based relations are more stable across counties. This suggests the paramount importance of form in the implementation of planning policy. What is clear is that it is geometrical relations which exhibit the greatest diversity, and that such relations should be incorporated into classifications of settlements *vis-à-vis* planning policy. More detailed interpretations are possible, but these must await further research and analysis and more meaningful classifications of settlements with respect to both morphology and planning policy.

In this chapter, we have been content simply to develop descriptive measures of settlement form based on standard methods of scaling and dimensionality which underpin the study of morphology, through allometry and fractal geometry. We have not implied, in any sense, that settlement forms which are characterized by particular dimensions indicating their density and space-filling properties, provide any indicator of their optimality or efficiency. In fact, one of the most controversial issues in the study of urban form has been over questions of whether very different forms such as linear versus concentric, high versus low density, radial versus grid, are more optimal than one another. For example, from the point of view of transport accessibility, indices can be derived which show that these various forms all embody some ideal attributes of such accessibility. Questions of optimal urban form from the point of view of energy use also provide contradictory conclusions depending upon what measures are constructed. Moreover in this context, it could be argued that Green Belt Policy has both increased the journey to work at a cost but increased access to the countryside as a benefit, and so on and so forth. In future studies, we might address these issues, but we feel that at this point that we have at least provided a rich source of suggestions which might condition future research, which involves a reworking and extension of the ideas presented here.